

## Economics 150, Intermediate Microeconomics, Fall 2008

*Please put your **discussion-section time** and **TA name** on your answer.*

Homework 4, Wednesday, October 8  
Due Wednesday, October 15

This homework first examines the notion of consumer surplus. Consider the utility function  $U(x_1, x_2) = x_1x_2$ .

- 4.1 Find the demand function for good  $x_1$  and the income elasticity of good  $x_1$ .
- 4.2 Find the indirect utility function.
- 4.3 Find the compensated demand function for good  $x_1$ .
- 4.4 Let  $I = 10$  and  $p_1 = p_2 = 1$ . Now consider the effects of increasing the price of good 1 to  $p_1 = 4$ . Find the compensating variation, the equivalent variation, and the change in consumer surplus associated with this price increase. Explain why these differ, and why they bear the relationship they do to one another.
- 4.5 Now repeat your answers to question [4.1] and [4.3]–[4.4], this time with utility function  $U(x_1, x_2) = 10x_1 - x_1^2 + 10x_2$ . Explain why the compensating variation, equivalent variation, and change in consumer surplus agree in this case. Why didn't you need the indirect utility function in this case? Use the differing income elasticities you have derived for the two utility functions to explain the difference in your answers to [4.4] and [4.5].
- 4.6 Suppose you are a judge hearing an antitrust case, and have concluded that the firms in this case illegally colluded to set the price  $p_1$  of their output higher than the "market" price  $p'_1$ . You now must determine the fine to impose on these firms. Your goal is to charge a fine equal to the harm caused to consumers by the excessively high price. Would the concepts of compensating variation, equivalent variation, and change in consumer surplus help you fix this fine? If you could calculate each of these, which one would you use? Which of these would you most likely be able to calculate, and how would you go about doing so?

We now briefly explore the connection between preferences and utility. Suppose first that Alice consumes apples (good 1) and bananas (good 2), and prefers consumption bundle  $x$  to bundle  $x'$  if the total of apples and bananas in the former is higher. Hence, Alice does not care about the mix of apples and bananas she consumes, only the total.

- 4.7 Show that Alice's preferences are complete, transitive, and monotonic. Given an intuitive explanation (a precise argument is rather tedious, so intuition will do) of why they are continuous.
- 4.8 Show that the utility function  $u(x_1, x_2) = x_1 + x_2$  captures these preferences.
- 4.9 Show that the utility function  $u(x_1, x_2) = x_1 + x_2$  is quasiconcave, but that these preferences do not satisfy the strict definition of convexity that we offered in class. You can conclude that this definition of convex preferences is sufficient for the utility function to be quasiconcave, but not necessary, with the case of perfect substitutes being the exception. Suggest a weaker definition of convex preferences and argue that it is equivalent (i.e., necessary and sufficient) to quasiconcavity of the utility function.

Now suppose that Alice likes to eat apples and bananas in pairs, and hence prefers consumption bundle  $x$  to  $x'$  if the number of pairs of apples and bananas in  $x$  exceeds that in  $x'$  (or equivalently, if the minimum of the number of apples and bananas in bundle  $x$  exceeds the minimum in bundle  $x'$ ).

- 4.10 Show carefully that Alice's preferences are complete, transitive, and convex. Explain intuitively why they are continuous.
- 4.11 Show that the utility function  $u(x_1, x_2) = \min\{x_1, x_2\}$  captures these preferences.
- 4.12 Show that these preferences do not satisfy the definition of monotonicity that we offered in class. You can conclude that preference exhibiting perfect complementarity does not satisfy our monotonicity assumption. Suggest a weaker definition of monotonicity and argue that this definition is satisfied in the case of perfect complements. (Think carefully about what it means for a definition of monotonicity to be weaker. In particular, a weaker definition of monotonicity would require a stronger relationship between bundles  $x$  and  $x'$  in order to conclude that  $x$  is preferred to  $x'$ .)